PHYS 2211

Look over
Chapter 9 Sections 1-12
Examples: 1, 4, 5, 6, 7, 8, 9, 10,

PHYS 1111

Look over
Chapter 7 Sections 1-8, 10
examples 2, 3, 4, 6, 7, 8,9, 10 and 11

How To Make Physics Pay

We will now look at a way of calculating where the pool balls will go. To do this we will also have to use the Law of Conservation of Energy

The Center of Mass

So far we have been treating objects as particles, having mass but no size.

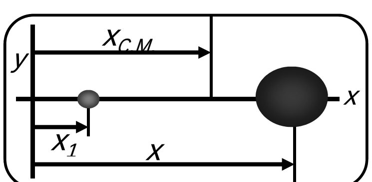
This is fine for transnational motion, where each point on an object experiences the same displacement.

But even when an object rotates or vibrates as it moves, there is one point on the object, called the Center of Mass, that moves in the same way that a single particle subject to the same force would move.

The Center of Mass

We can balance the teeter-totter at the point that we could replace all the mass with just a particle. This is the center of mass or sometimes called the center of gravity.

Center of Mass 1-D



We define the center of mass for a system of two particles as:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Where: x_{CM}=The position of the Center of Mass along the x-axis.

x_1=The position of particle #1
x_2=The position of particle #2
x_1=The mass of particle #1
x_2=The mass of particle #2

Center of Mass for Many Particles

For n particles:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \ldots m_n x_n}{m_1 + m_2 + \ldots m_n} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

The Center of Mass 3-d

In three dimensions we just need to find the location of the Center of Mass for all 3 coordinates (*x,y,z*) separately by:

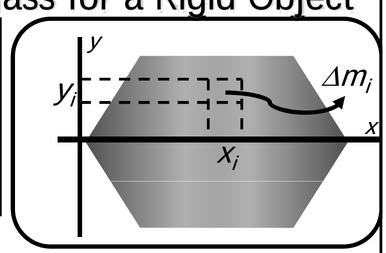
$$x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{M} \quad y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{M} \quad z_{CM} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

In vector notation the center of mass can be written as:

$$\vec{r}_{CM} = r_{x_{CM}}\,\hat{i} + r_{y_{CM}}\,\hat{j} + r_{z_{CM}}\,\hat{k}$$

The Center of Mass for a Rigid Object

The number of particles (atoms) in a rigid object is so large and their spacing so small that we can treat such an object as though it had a continuous distribution of mass.



We can subdivided the object into n small elements of mass Δm_i located approximately at (x_i, y_i, z_i)

$$x_{CM} \approx \frac{\sum_{i=1}^{n} \Delta m_i x_i}{M} \quad y_{CM} \approx \frac{\sum_{i=1}^{n} \Delta m_i y_i}{M} \quad z_{CM} \approx \frac{\sum_{i=1}^{n} \Delta m_i z_i}{M}$$

Center of Mass Equations

Now let the elements of mass be further subdivided so that the number of elements n tends to infinity:

$$x_{CM} = \lim_{\Delta m_i \Rightarrow 0} \frac{\sum_{i=1}^{n} \Delta m_i x_i}{M} \qquad x_{CM} = \frac{\int x\, dm}{M}$$

$$y_{CM} = \lim_{\Delta m_i \Rightarrow 0} \frac{\sum_{i=1}^{n} \Delta m_i y_i}{M} \qquad y_{CM} = \frac{\int y\, dm}{M}$$

$$z_{CM} = \lim_{\Delta m_i \Rightarrow 0} \frac{\sum_{i=1}^{n} \Delta m_i z_i}{M} \qquad z_{CM} = \frac{\int z\, dm}{M}$$

Motion of the Center of Mass

If you roll a cue ball at a 2nd billiard ball that is at rest, we expect that the two balls will roll forward after impact.

You would be surprised if both balls started rolling back at you or if the balls rolled off at a right angle to the original motion.

We are use to the fact that the Center of Mass of the two balls moves forward as if no collision had happened at all.

Newton's 2nd Law for a System of Particles

Although the center of mass is just a point, it moves like a particle whose mass is equal to the total mass of the system and we can assign a position, a velocity and an acceleration to it.

The equation of motion for the center of mass is:

$$\sum_i \vec{F}_i = \vec{F}_{Ext} = M\vec{a}_{CM}$$

Where F_i is the vector sum of all the forces acting on the system. The forces exerted by one part of the system on the others cancel each other out by Newton's 3th Law.

Linear Momentum of a Particle

The **Linear Momentum** of a particle is a measure of how hard it will be to stop the particle. It is defined as the product of the mass and the velocity

$$\vec{p} = m\vec{v}$$

The unit for Linear Momentum is: $\frac{kg\ m}{s}$

Momentum and Newton's 2nd Law

In terms of Linear Momentum Newton's second law reads:

The rate of change of Linear Momentum of a body is proportional to the resulting force acting on the body and is in the direction of that force.

$$F = \lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \frac{dp}{dt}$$

Linear Momentum of a System of Particles

Instead of a single particle if we have a system of n particles, with masses (m_1, m_2,...m_n). The particles in the system may interact with each other and there may be external forces acting on the system as well.

Each particle will have a velocity and a Linear Momentum :

$$p_1 = m_1 v_1,\ p_2 = m_2 v_2, \ldots\ p_n = m_n v_n$$

The total Linear Momentum for the system will be:

$$p_{Total} = p_1 + p_2 + \ldots + p_n$$
$$= m_1 v_1 + m_2 v_2 \ldots + m_n v_n$$

Which by our definition of the center of mass becomes:

$$p_{Total} = M v_{CM}$$

The total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of the Center of mass.

Conservation of Linear Momentum

From Newton's 2nd Law in terms of Linear Momentum we have for a system of particles:

$$\vec{F}_{Ext} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}_{Total}}{\Delta t} = \frac{d\vec{p}_{Total}}{dt}$$

If there are no External Forces then we get:

$$\lim_{\Delta t \to 0} \frac{\Delta \vec{p}_{Total}}{\Delta t} = 0$$

or

$$\vec{p}_{Total} = \text{a constant}$$

Conservation of Linear Momentum

$$\vec{p}_{Total} = \text{a constant}$$

This is the **Principle of the conservation of Linear Momentum**.

The Linear Momentum of the individual particles may change, but their sum remains constant if there are no net external forces.

The law of Conservation of Linear Momentum holds true even in atomic and nuclear physics, although Newtonian Mechanics does not. Hence this conservation law must be even more fundamental then Newtonian Mechanics.

Conservation Principles

The Conservation of Linear Momentum Principle is the second of the great conservation principles that we have met so far, the first being the conservation of Energy principle.

Conservation principles are of theoretical and practical importance in physics because they are simple and universal

Different observers, each in his or her own reference frame would all agree, if they watch the same changing system, that the conservation laws applied to system.

Example 1

1)A railroad car moves at a constant speed of 3.20 m/s under a grain elevator. Grain drops into it at a rate of 540 kg/min. What force must be applied to the railroad car, in the absence of friction, to keep it moving at a constant speed?

Collisions

We can learn about objects of all kinds by observing them as they collide with each other.

Objects of interest that we study by watching collisions range from subatomic particles whose masses are $\approx 10^{-27}$kg to galaxies, whose masses are on the order of $\approx 10^{27}$kg and everyday objects with masses in-between.

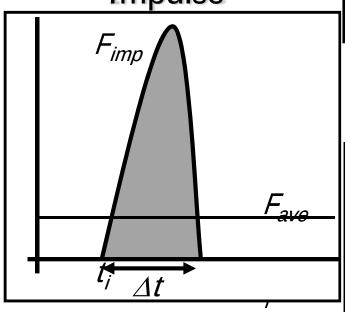
Before and After

The principal tools for analyzing collisions are the laws of Conservation of Energy and Momentum.

In a collision a relatively large force acts on each of the colliding particles for a relatively short time.

The basic idea of a collision is that the motion of the colliding particles can be separated in time into ***"before the collision"*** and ***"after the collision"***

Impulse



Assume that during a collision F_{imp} acts from t_i until t_f then:

$$F_{ave} = \frac{\Delta p}{\Delta t} \Rightarrow$$
$$\Delta p = F_{ave}\,\Delta t = I_{imp}$$

Where I_{imp} is the **Impulse** which is a vector that points in the direction of the vector change in momentum and is equal to the area under the force time curve.

The Impulse is a measure of the strength and duration of the collision force.

Impulse

If the collision force is large compared to the external forces and the collision force acts for a short time then the momentum for the system of particles is conserved.

In this case we can then say that the momentum of a system of particles just before the particles collide is equal to the momentum of the system just after the particles.

Two Kinds of Collisions

Collisions are usually classified according to whether of not Kinetic Energy is conserved in the collision.

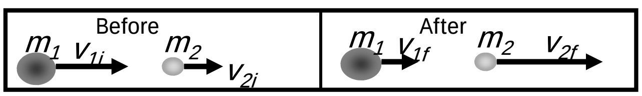
1) **Elastic Collisions-** Kinetic Energy is conserved. In these collisions the objects do not “Stick Together” at all.

2) **Inelastic Collision**- Kinetic Energy is not conserved. In these collisions the objects do “Stick Together” some what.

A completely inelastic collision is when two objects completely stick together.

All collision between gross objects are inelastic to some extent.

Collisions in 1-Dimension



	Elastic Collisions 1-D
p	$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
KE	$(1/2)m_1 (v_{1i})^2 + (1/2)m_2 (v_{2i})^2 = (1/2)m_1 (v_{1f})^2 + (1/2)m_2 (v_{2f})^2$

	Completely Inelastic Collisions 1-D
p	$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

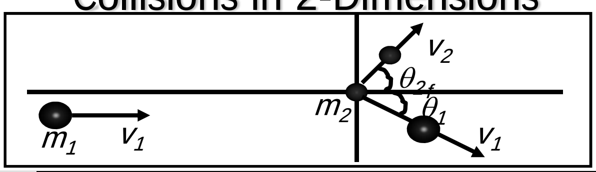
Example 2

2)Two titanium spheres approach each other head-on with the same speed and collide elastically. After the collision, one of the spheres, whose mass is *300 g*, remains at rest. What is the mass of the other sphere.

Example 3

3)Meteor Crater in Arizona is thought to have been formed by the impact of a meteor with the Earth some 20,000 yrs ago. The mass of the meteor is estimated at 5×10^{10}kg, and its speed at 7200 m/s. What speed would such a meteor impart to the Earth in an head-on collision?

Collisions in 2-Dimensions



	Elastic Collisions 2-D
p_x	$m_1v_{1i}=m_1v_{1f}\cos\theta_1+m_2v_{2f}\cos\theta_1$
p_y	$0=m_1v_{1f}\sin\theta_1+m_2v_{2f}\sin\theta_2$
KE	$(1/2)m_1(v_{1i})^2=(1/2)m_1(v_{1f})^2+(1/2)m_2(v_{2f})^2$

	Completely Inelastic Collisions 2-D
p_x	$m_1v_1=(m_1+m_2)v\cos\theta$
p_y	$0=(m_1+m_2)v\sin\theta$

Example 4

4)Two cars collide at an intersection. Car 1 has a mass of *1200 kg* and is moving at a velocity of *95.0 km/hr* due east and car 2 has a mass of
1400 km/hr due north. The cars stick together and move off as one at angle θ.
a)What is the angle θ?
b)What is the final velocity of the combined cars.

Summary of Chapter 9

- Momentum of an object: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$.
- Newton's second law: $\Sigma\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$

•Total momentum of an isolated system of objects is conserved.

- During a collision, the colliding objects can be considered to be an isolated system even if external forces exist, as long as they are not too large.
- Momentum will therefore be conserved during collisions.

Summary of Chapter 9, cont.

- $\text{Impulse} = \vec{\mathbf{F}}\,\Delta t = \Delta\vec{\mathbf{p}}$
- In an elastic collision, total kinetic energy is also conserved.
- In an inelastic collision, some kinetic energy is lost.
- In a completely inelastic collision, the two objects stick together after the collision.
- The center of mass of a system is the point at which external forces can be considered to act.