PHYS 2211

Look over:
Chapter 3 Sections 1-8
Examples 2, 3, 4, 5, 6, 7, 8 and 9

PHYS 1111

Look over:
Chapter 1 Sections 7 and 8
Examples 5, 6, 7 and 8

Topics Covered

1) What are Vectors and Scalars.
2) Two ways of specifying Vectors: Components or Angle and Magnitude.
3) How to go from one description of a vector to another.
4) Two ways of add Vectors: Graphical and Analytical.
5) Unit Vectors.

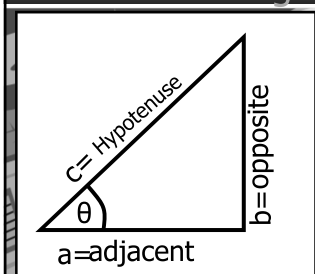
Vectors and Scalars

A **Scalar** is a number that only has a Magnitude associated with it.

A **Vector** is any quantity that needs two things (a **size** and a **direction**) to completely describe it.

The size of a Vector is referred to as its **Magnitude**

The Right Triangle



$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

The Pythagorean theorem relates the 3 sides of a right triangle as:

$$c^2 = a^2 + b^2$$

Vectors

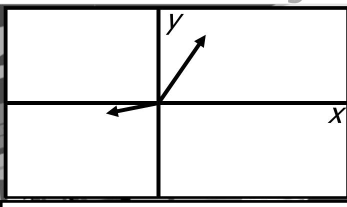
To indicate that a quantity is a vector we draw an arrow over its symbol or write it in boldface.

$\vec{a}$ or $\mathbf{a}$

When drawing vectors on a set of *x* and *y* coordinates we represent the vectors as rays (i.e. a line with an arrow at the end of it).

The length of the ray corresponds to the magnitude of the vector. The direction that the ray points is the direction of the vector.

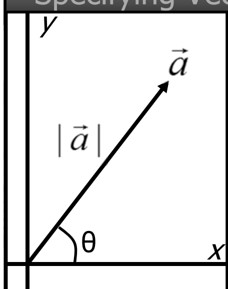
Drawing Vectors



In drawing vectors we have the freedom to place them any where we want on the coordinate system as long as we do not change the direction or magnitude of the vector.

Any two vectors are equal if and only if their directions and the their magnitude are equal.

Specifying Vectors: Magnitude and Angle

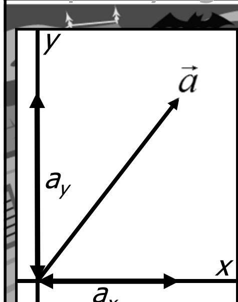


The first way to specify a vector is to give both the direction of the vector and the magnitude. This is sometimes called **Polar Coordinates**.

In most cases the direction is given in terms of an angle θ measured from the (+) x axis.

The magnitude is given in units appropriate to what the vector represents and is represented by the symbol of the vector written between two vertical lines. $|\vec{a}|$

Specifying Vectors: Components

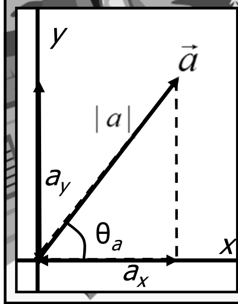


Another way to specify a vector is state the x, and y coordinates of the tip of the vector. Then a ray can be drawn from the origin of the coordinate system to the point given by the coordinates.

The coordinates of the tip of the vector are called the components of the vector and are written as:

$$\vec{a} = (a_x, a_y)$$

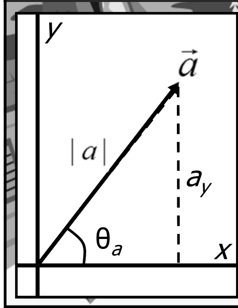
How to Go From Magnitude and Angle to Components



If you know the magnitude and the angle you can find the components using:

$$a_x = |a| \cos \theta_a$$
$$a_y = |a| \sin \theta_a$$

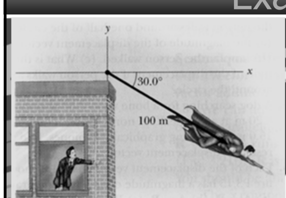
How to Go From Components to Magnitude and Angle



If you know the components you can find the magnitude and the angle using:

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$
$$\tan \theta_a = \frac{a_y}{a_x}$$

Example 1



1)Find the horizontal and vertical components of the *100 m* displacement of a superhero who flies from the top of a tall building following the path shown above.

Example 2

2)The x component of a certain vector is *-20.0* units and the y component is *+30.0* units.
a)What is the magnitude of the vector?
b)What is the angle between the direction of the vector and the positive *x* axis?

Vector Addition

There will be times when we will need to add together two or more vectors. Vectors do not add like regular numbers (scalars). Some of the properties of vector addition are:

1)**Addition**-Two vectors added together will give you a new vector.

$$\vec{r} = \vec{a} + \vec{b}$$

2)**Commutative Law**: If you add vector ***a*** to vector ***b***, you will get the same vector that you will get if you had add vector ***b*** to vector ***a***.

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

Vector Addition

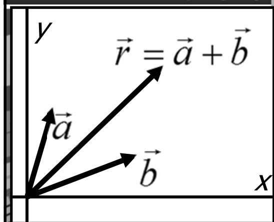
3)**Associative Law**:If add together vector ***a*** and vector ***b*** and then add a vector ***c*** this is the same as adding together vector ***b*** and vector ***c*** and then adding vector ***a***.

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

4)**Subtraction**:Subtracting vector b from vector a is the same as adding negative one times b to a.

$$\vec{a} - \vec{b} = \vec{a} - 1(\vec{b})$$

The Graphical Method



The first way is the **Graphical Method**.

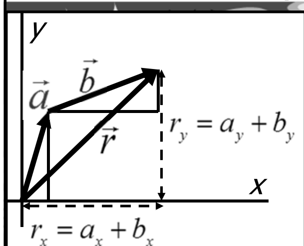
To add vectors using the graphical method:

1)Draw the first vector to scale with the tail of the vector at the origin.

2)Draw the second vector to scale with its tail at the tip of the first vector.

3)The sum of the two vectors is now the vector that connects the tail of the first to the tip of the second.

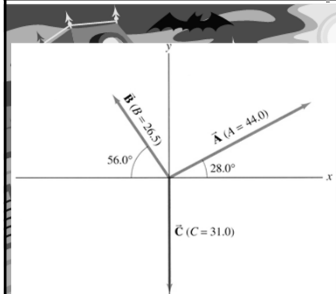
The Analytical Method



The analytical method consists of adding up the x components of the two vectors to get the x component of the new vector. Then add up the y components of the two vectors to get the y component of the new vector.

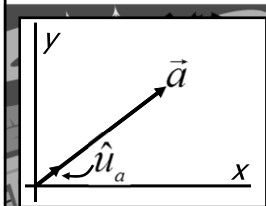
$$r_x = a_x + b_x$$
$$r_y = a_y + b_y$$

Example 3



4) Three vectors are shown above. Determine the sum of the three vectors. Give the result in terms of a) components,
b) magnitude and angle.

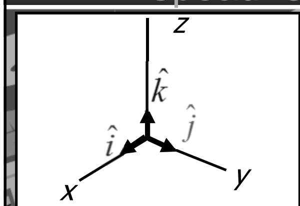
Unit Vectors



For any vector we can define a unit vector by dividing the vector by its magnitude. The magnitude of a unit vector is one.

$$\hat{u}_a = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_x^2 + a_y^2}}$$

Special Unit Vectors



$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

We can write an vector in terms of three unit vectors which are in the directions of the *X*, *Y* and *Z* axis and the components of the vector.

Example 4

5)A displacement vector lying in the *x-y* plane has a magnitude of *50.0 m* and is directed at an angle 0f *120°* to the positive x axis. Find the the *x* and *y* components of this vector and express the vector in unit vector notation.

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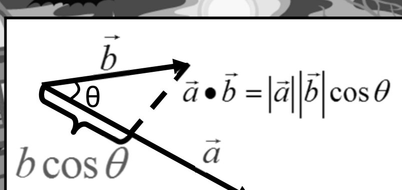
Vector Multiplication

The **Scalar Product** (or **Dot Product**) of any two vectors ***a*** and ***b*** is a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle between them:

$$\vec{a} \bullet \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

What the Dot Product Tells Us



The Dot Product is the product of the magnitude of vector ***a*** and the projection of vector ***b*** onto ***a***.

Properties of the Dot Product

1)The Dot Product is Commutative:

$$\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$$

2)The Dot Product is Distributive:

$$\vec{a} \bullet \left(\vec{b} + \vec{c}\right) = \vec{a} \bullet \vec{b} + \vec{a} \bullet \vec{c}$$

The Dot Product and Unit Vectors

For the Units Vectors we find:

$$\hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1$$

$$\hat{i} \bullet \hat{j} = \hat{i} \bullet \hat{k} = \hat{j} \bullet \hat{k} = 0$$

If we write a vector in unit vector form:

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

We can find another way of calculating the Dot Product:

$$\vec{a} \bullet \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

The Vector Product

The Vector (or Cross Product) for any two vectors ***a*** and ***b*** is a third vector ***c***, the magnitude of which is given by:

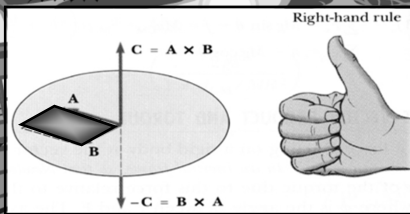
$$|\vec{a}||\vec{b}|\sin\theta$$

and a direction that is parallel to the plane formed by the vectors ***a*** and ***b***.

$$\vec{c} = \vec{a} \times \vec{b}$$

$$|\vec{c}| \equiv |\vec{a}||\vec{b}|\sin\theta$$

What the Cross Product Tells



The magnitude of the cross product of two vectors is equal to the area of the parallelogram from by the two vectors.

Properties of the Cross Product

1)The Cross Product is **not** Commutative:

$$\vec{a} \times \vec{b} = -\left(\vec{b} \times \vec{a}\right)$$

1)The Cross Product is Distributive:

$$\vec{a} \times \left(\vec{b} + \vec{c}\right) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

The Cross Product and Unit Vectors

For the Units Vectors we find:

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$$

$$\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$$

Another Way to Calculate the Cross Product

If we write a vector in unit vector form:

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

We can find another way of calculating the Cross Product:

$$\vec{a} \times \vec{b} = \left(a_y b_z - a_z b_y\right)\hat{i} + \left(a_z b_x - a_x b_z\right)\hat{j} + \left(a_x b_y - a_y b_x\right)\hat{k}$$

Example 5

6)Given the two vectors:

$$\vec{a} = 2\hat{i} + 3\hat{j}$$

$$\vec{b} = -\hat{i} + 2\hat{j}$$

a) find their Dot product
b) find their Cross product.

Summary of Vectors

- A quantity with magnitude and direction is a vector.
- A quantity with magnitude but no direction is a scalar.
- Vector addition can be done either graphically or using components.
- The sum is called the resultant vector.